(Pages : 4) **4645**

Reg. No. : ....

Name : ....

Fifth Semester B.Tech. Degree Examination, September 2014
(2008 Scheme)
(Special Supplementary)
08.501 : ENGINEERING MATHEMATICS – IV
Complex Analysis and Linear Algebra (T A)

Time: 3 Hours Max. Marks: 100

Instruction : *Answer **all** questions from Part – **A** and **one full** question from **each** Module of Part – **B**.*

PART – A

1. Show that f(z) = Re z is continuous but not differentiable.

2. Show that the function $u = 2xy + 3xy^2 - 2y^3$ can be the real part of an analytic function. Find its harmonic conjugate.

3. If f(z) = u + iv is analytic then prove that the families of curves. u = constant and v = constant are orthogonal trajectories.

4. Find the image of x > 1, y > 0 under $w = \frac{1}{z}$.

5. Show that for every path between the limits

$$\int_{-2}^{-2+i} (2+z)^2\, dz = \frac{-i}{3}.$$

6. Evaluate f(2) and f(3) where

$$f(a) = \int_C \frac{(2z^2 - z - 2)}{z - a}\, dz$$ and C is the circle | z | = 2.5.

P.T.O.

7. Find the nature and location of the singularities of $\frac{\tan z}{z}$.

8. Define a vectorspace. Give an example and verify it.

9. Let $y = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$ and $u = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$. Find the orthogonal projection of y on u.

10. Let A be an n × n matrix which is symmetric. Prove that the quadratic form $x^T A x$ is negative definite if and only if the eigen values of A are all negative.

(10×4=40 Marks)

PART – B

(Answer **one** question from **each** Module. **Each** question carries **20** marks).

Module – I

11. a) Prove that $f(z) = \frac{x^3 y (y - ix)}{x^6 + y^2}$ for $z \neq 0$ and $f(0) = 0$ is not continuous at origin.

b) Show that $f(z) = \log z$ is differentiable except at $z = 0$. Also find its derivative.

c) Determine the region of the w-plane into which the region bounded by $x = 1$, $y = 1$ and $x + y = 1$ is mapped by $w = z^2$.

12. a) Determine the analytic function $f(z) = u + iv$ if $u - v = \frac{\cos x + \sin x - e^{-y}}{2(\cos x - \cosh 2y)}$ and $f\left(\frac{\pi}{2}\right) = 0$.

b) Prove that an analytic function with constant argument is constant.

c) Find the bilinear transformation which maps the points $z = 1, i, -1$ onto the points $w = i, 0, -i$. Also find the image of $|z| < 1$.

Module – II

13. a) Evaluate $\oint_C \frac{e^{3z}dz}{(z - \ln 2)^4}$ where C is the square with vertices at ± 1 and $\pm i$.

b) Evaluate $\int_C \frac{e^z}{z(1-z)^3} dz$ where C is $|z-1| = \frac{1}{2}$.

c) Expand $\frac{1}{(z+1)(z+3)}$ in Laurent series valid for $1 < |z| < 3$.

14. a) Evaluate $\int_C \frac{z \sec z}{(1-z)^2} dz$ where C is $|z| = \frac{3}{2}$.

b) Evaluate $\int_0^{2\pi} \frac{1}{1 - 2x\sin\theta + x^2} d\theta$; $0 < x < 1$.

c) Evaluate $\int_0^{\infty} \frac{1}{1+x^4} dx$.

Module – III

15. a) Let $b_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $b_2 = \begin{bmatrix} -3 \\ 4 \\ 0 \end{bmatrix}$ and $b_3 = \begin{bmatrix} -8 \\ 2 \\ 3 \end{bmatrix}$. Show that $\mathcal{B} = \{b_1, b_2, b_3\}$ is a basis of $\mathbb{R}^3$. Find the change of coordinates matrix from $\mathcal{B}$ to the standard basis of $\mathbb{R}^3$.

b) Find the LU decomposition of

$$\begin{bmatrix} 2 & 4 & -1 & 5 & -2 \\ -4 & -5 & 3 & -8 & 1 \\ 2 & -5 & -4 & 1 & 8 \\ -6 & 0 & 7 & -3 & 1 \end{bmatrix}$$

c) Find the maximum value of $\phi(x) = 5x_1^2 + 5x_2^2 - 4x_1x_2$ subject to the constraint $x^T x = 1$. Also find a unit vector where this maximum is attained.

16. a) Let $A = \begin{bmatrix} 1 & -3 & -4 \\ -4 & 6 & -2 \\ -3 & 7 & 6 \end{bmatrix}$ and $b = \begin{bmatrix} 3 \\ 3 \\ -4 \end{bmatrix}$. Determine whether b is in the column space of A. **20**

b) Explain Gram-Schmidt orthogonalization process. Use this to find an orthogonal basis for the column space of the matrix

$$\begin{bmatrix} 3 & -5 & 1 \\ 1 & 1 & 1 \\ -1 & 5 & -2 \\ 3 & -7 & 8 \end{bmatrix}.$$

c) Construct a singular value decomposition of $A = \begin{bmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{bmatrix}$.
